Comments on Real Options Exam Results LIS 10 June 2020

Generally, the quality of some of the exams is impressive, and most students provided appropriate answers. The average exam grades are somewhat above last year, well done. Some students did much better on the exam than on coursework, some did not. Below are some of the errors of a few students. There are some students who could probably benefit from taking any re-sit.

Q1

1. Most students provided an extensive list of assumptions, also stating that this appears to be a perpetual American call option with constant K, yield and volatility.
2. The basic calculations are relatively easy, so those who excelled also showed the ODE, and in some cases the Delta and Gamma results that solve the ODE. The ROV is also the value of the PUD.
3. Curiously some students multiplied V by 10% to get V, so calculated V-K=-8.7. Why?
4. Some students calculated β_1 incorrectly, errors which could be corrected in the re-sit through practicing exercises.
5. Interpretation is that this is an in-the-money development option, where the ROV appears to be embedded in the PUD. Some students noted that the ROV is very sensitive to changes in r, δ and σ, so this primary "asset" could be quite risky. Many students noted that the assumptions given under 1 are not necessarily realistic, and that some parameter values might be hard to calibrate accurately. But some students gave limited interpretations.
6. Some students did not calculate V* if σ_{NG}=3%, others provided answers similar to column D below.

	A	B	C	D
1	**Q1 Perpetual American Option**			
2				
3	V	13,000,000		13,000,000
4	K	10,000,000		10,000,000
5	σ	0.30		0.03
6	r	0.05		0.05
7	δ	0.04		0.04
8	ROV	5,646,677	IF(B3<B12,(B12-B4)*(B3/B12)^B14,B10)	3,000,000
9	ROV	5,646,677	IF(B3<B12,B13*(B3^B14),B10)	3,000,000
10	V-K	3,000,000	B3-B4	3,000,000
11	F'(V)	0.66	IF(B3<B12,B13*B14*(B3^(B14-1)),1)	1.00
12	V*	29,514,843	(B14/(B14-1))*B4	12,999,998
13	A	0.00010	(B12-B4)/(B12^B14)	0.00
14	β_1	1.5124	0.5-(B6-B7)/(B5^2)+SQRT(((B6-B7)/(B5^2)-0.5)^2 + 2*B6/(B5^2))	4.3333
15				
16	ODE	0.00	0.5*(B5^2)*(B3^2)*B17+(B6-B7)*B3*B11-B6*B9	-20000.00
17	F''(V)	0.00	IF(B3<B12,B13*B14*(B14-1)*(B3^(B14-2)),0)	0.00
18	F'(V*)	1.00	B13*B14*(B12^(B14-1))	1.00
19	F(V*)	19514842.76	IF(B3<B12,B13*(B12^B14),B10)	3000000.00
20	V*-K	19514842.76	B12-B4	2999997.69
21	V/K	1.30		1.30
22				
23	Q A	5,646,677	ROV	
24	Q B	29,514,843	or 2.2 times current V	
25	Q D	12,999,998	or almost SWN's V/K NPV rule	

7. Should Bill hedge and then drill following both his Rule and RO theory? Would this hedge be feasible using the entire term structure of NG futures prices, given production uncertainty?

8. The ODE is solved (B16) using delta and gamma in column B, but not in column D (which indicates the option should have been already exercised.) B19=B20 show that the value matching conditions hold at V*.

Q2

1. Only five students addressed this question, perhaps because it was somewhat different from the switching shut-down option in the tutorial.

2. Although the average grade was similar to other questions, the requirement to switch outputs (which are converted from values by multiplying by the asset or convenience yield) is an extra task, introducing another possible error.

3. Note that this is not a typical solution to a quadratic equation governing a function of homogeneity of degree one. The analytical solution is supposed to be simple, and did not seem to be a problem for the RRC group.

Q3 This seemed to me an easy and relevant question, especially given the hypothetical "market price of debt". None of the frackers are 100% leveraged (excepting maybe GPOR, with a current market-to-market PV10 of proven reserves), so the zero-accounting equity was posed as an extreme.

	A	B	C	D	E	F
1		**Q3 Real Debt Option**				
2						
3						
4		**ACCOUNTS**	**REAL**			
5	ASSETS	2,000,000,000	1,918,658,475	C10-C15*C19*(C19/C10)^-C20		
6	DEBT	2,000,000,000	1,201,598,043	B6+((1-C15)*C19-B6)*(C19/C10)^-C20		
7	EQ	0	**717,060,432**	C5-C6		
8						
9	INPUT					
10	NOM V	2,000,000,000	2,000,000,000			
11	Coupon	80,000,000	80,000,000			
12	NOM Debt	2,000,000,000	2,000,000,000			
13	δ		0.04			
14	σ		0.30			
15	α		0.20			
16	τ		0.00			
17	r(f)		0.04			
18	OUTPUT					
19	V_B		723,828,411	(1-C16)*(C11/C17)*(-C20/(-C20+1))		
20	β_2		-0.567			
21						
22	β_2	(0.5-(C17-C13)/(C14^2)-SQRT((0.5-(C17-C13)/(C14^2))^2+2*C17/(C14^2)))				
23						
24	Q A	723,828,411	36% of current value, real constraint is paying $80m per year to keep real equity call option alive			
25	Q B	0.832225057	debt is still undervalued in the market, so consider repurchasing			
26	Q C	717,060,432	real equity value is substantial, while accounting value is nil			
27	Q's	consider selling V to repurchase debt, and equity if much less than real equity value				

1. Without NG volatility this firm is on the verge of insolvency, but with high volatility this equity is quite valuable as a real call option, maintained by paying $80m interest every year, which may be a problem. Some students calculated β_2 incorrectly, some students calculated D* incorrectly, most students observed that this Leland (1994) model solves an ODE where the contingent claim is debt.

2. "Should Roby try to repurchase debt if it can be repurchased in the market at half of the nominal value D?" The risky debt is worth $1.2B according to the model, so repurchasing $2B nominal for $1B cash would produce the results below.

	G	H	I
4		**ACCOUNTS**	**REAL**
5	ASSETS	1,000,000,000	1,000,000,000
6	DEBT	0	0
7	EQ	1,000,000,000	**1,000,000,000**

Both real and accounting equity would increase, so this appears to be an obvious strategy, if debt prices remained depressed. But where would Roby get $1B cash (by selling proven reserves worth that amount, in a distressed market?). Are debtholders dumb or realistic? Many of these frackers did repurchase debt during the 2Q 2020, which some CFO/CEOs noted as an opportunity.

3. No equity prices are given in this question, but repurchasing equity at lower than the real equity value might be considered. With 100% leverage this strategy is rather risky, naturally.

Q4. 1. Only five students answered this question, reflecting what she/he might have learned in doing a group project, which could be improved by more time, money and information. The best student indicated paying attention to the webinar and development of NG prices since the project submission, and suggesting that drilling and repurchasing strategy should be dynamic.

2. Working inside any of these firms, there would be available information on projected production, precise hedging details and possibilities, and possibilities and costs for turning PD into cash, and prices for repurchasing debt and equity daily.

You may have noticed that share prices for all of your project companies declined as U.S. natural gas prices fell after your project submissions, and both NG and NGL prices have been volatile and with changing correlations. It will be interesting to see in the Q3 results which companies have taken advantage of the apparent opportunity for repurchasing debt and equity; several CEOs mentioned this opportunity in the Q2 webinars, and probably wished they had more spare cash and less debt.

COMBINED PROJECT AND EXAM GRADES

Some students who did well on their project, did well on the exam showing performance persistence, but there were some disappointing reversals. Those who did well provided complete sets of theory/ assumptions, correct results, and interesting business interpretations. Those who did poorly sometimes provided incorrect results in at least one question, showing that the errors were perhaps due to insufficient independent practice in doing exercises. Probably more in-class exercises are warranted (and could be interesting). Also focused online discussions and live exercises will be considered in the future, with or without the virus.

Although the focus in this class this year is on firms that disclose lots (but not everything) about asset composition and value, similar questions might be posed regarding zombie Portuguese companies, or many distressed European companies in connection with the virus. Airlines, hotels, universities, some urban real estate, retailers, and many other sectors are possible candidates for applying these particularly relevant real option theories during these times. Have you learned more in this experience than anticipated? Well done, especially given the disruption due to the pandemic.